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BRIEF COMMUNICATIONS

TWO-PHASE FLOW IN BENDS

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Abstract—An equation is developed for use in predicting the two-phase multiplier for pressure drop in bends; the equation simplifies the use of an existing method. The method is also compared for the first time with data at high density ratios ($\rho_L/\rho_G = 560$).

1. INTRODUCTION

In this note an existing method for predicting pressure drop in bends (Chisholm 1971) is developed so that the two-phase multiplier can be evaluated from a simple equation, avoiding the use of a graphical procedure. In addition the method is compared for the first time with data at higher density ratios ($\rho_L/\rho_G = 560$).

2. PRESSURE DROP ATTRIBUTABLE TO A BEND

The pressure drop attributable to a bend can be defined as

$$\Delta p_b = p_a - p_c + Dp_a z_a + Dp_c z_c, \quad [1]$$

where the subscripts a and c refer to properties at points a and c distant z_a and z_c upstream and downstream of the bend; p_a and p_c are static pressures at points a and c; and equilibrium flow exists at a and c and the static pressure gradients at these points are Dp_a and DP_c.

The pressure drop attributable to the bend in single-phase flow is frequently estimated using a resistance or pressure drop coefficient k defined by the equation

$$\Delta p_b = k\frac{G^2}{2\rho}. \quad [2]$$

3. AN ELEMENTARY MODEL

Romie, as quoted by Hoopes (1957), developed an equation which in terms of the momentum flux can be expressed

$$\frac{MF}{MF_{L0}} = \frac{x^2}{\alpha}\frac{\rho_L}{\rho_G} + \frac{(1-x)^2}{1-\alpha} \quad [3]$$

where

$$MF_{L0} = G^2/\rho_L. \quad [4]$$

This can be approximated (Chisholm 1971), except where the density ratio approaches unity, as

$$\frac{MF}{MF_{LO}} = 1 + \left(\frac{\rho_L}{\rho_G} - 1\right)\left\{\frac{1}{K}x(1-x) + x^2\right\}. \quad [5]$$

While [5] is not a precise transformation of [3] when the phase density ratio approaches unity, nevertheless [5] holds at the critical point.

For flow in a pipe of uniform cross-section, a change of momentum flux due to a change in velocity ratio K can therefore be expressed

$$\frac{\Delta(MF)}{MF_{LO}} = \left(\frac{\rho_L}{\rho_G} - 1\right)x(1-x)\Delta\left(\frac{1}{K}\right). \tag{6}$$

This equation of course assumes an incompressible and non-evaporating flow.

Let us assume that the bend separates the two phases with a resultant increase in the velocity ratio within the bend, then downstream of the bend there will be a further pressure loss as the momentum flux increases to the equilibrium value. Assume also that in single-phase flow the downstream effects are small, and the single-phase loss occurs within the bend, then the two-phase pressure drop, where the plane of the bend is in the horizontal plane, can be expressed as

$$\Delta p_b = \Delta p_{bLO}\phi^2_{LO} + \Delta(MF), \tag{7}$$

where the change of momentum flux is obtained from [6] with $\Delta(1/K)$ corresponding to the change in velocity ratio discussed above. Assume the two-phase multiplier within the bend is given with sufficient accuracy by homogeneous theory

$$\phi^2_{LO} = 1 + \left(\frac{\rho_L}{\rho_G} - 1\right)x. \tag{8}$$

From [2]

$$\Delta p_{bLO} = k_{LO}\frac{G^2}{2\rho_L}. \tag{9}$$

From [4] and [6]–[9]

$$\frac{\Delta p_b}{\Delta p_{bLO}} = 1 + \left(\frac{\rho_L}{\rho_G} - 1\right)\left\{Bx(1-x) + x^2\right\} \tag{10}$$

where

$$B = 1 + \frac{2}{k_{LO}}\Delta\left(\frac{1}{K}\right). \tag{11}$$

4. THE COEFFICIENT B

It has already been shown (Chisholm 1969) that the assumption that $\Delta(1/K)$ is a constant gives values of B following the general trend of experimental results for 90° bends, but overpredicts B at higher values of the pipe radius-to-diameter ratio (R/D).

Further examination has now suggested the following empirical equation

$$\Delta\left(\frac{1}{K}\right) = \frac{1.1}{2 + \frac{R}{D}}. \tag{12}$$

The magnitudes obtained from this equation are consistent with the model. From [11] and [12]

$$B = 1 + \frac{2.2}{k_{LO}\left(2 + \frac{R}{D}\right)}. \tag{13}$$

Table 1. Comparison of predicted and experimental B-coefficients for 90° bends

Test series	1	2	3	4	5	6	7
$\frac{R}{D}$	0	1.0	1.5	1.5	2.36	5.0	5.02
k_{LO}	1.25	0.310	0.174	0.282	0.250	0.234	0.300
B Experiment	1.8	3.4	4.5	3.4	Figure 1	2.4	Figure 1
Equation [12]	1.9	3.4	4.6	3.2	3.0	2.3	2.0
Author	1,2	1,2	1,2	1,2	3	1,2	3
Comments	Tee	Disturbance upstream		Disturbance upstream		k smaller in test 6 as higher Re	

Authors: 1—Chisholm (1971); 2—Fitzsimmons (1964); 3—Sekoda *et al.* (1969)

Table 1 compares previously reported values of B for 90° bends in the horizontal plane with predicted values using this equation; the agreement is to within 6 per cent. Equation [13] successfully correlates the data (test series 2 and 4) with an upstream disturbance (a change of section) which had not previously been satisfactorily correlated.

5. COMPARISON AT HIGH DENSITY RATIOS

Empirical values of B were previously obtained from data of Fitzsimmons (1964) with steam–water mixtures at pressures in excess of 55 bar ($\rho_L/\rho_G \leq 27.0$). The data of Sekoda *et al.* (1969) were obtained with air–water flow in 90° bends at 1.5 bar ($\rho_L/\rho_G \doteqdot 560$). These data therefore allow the procedure to be checked at higher density ratios.

The form of data presentation used by Sekoda *et al.* necessitates the use of the equation

$$\frac{\Delta p_b}{\Delta p_{bL}} = 1 + C\left(\frac{\Delta p_{bG}}{\Delta p_{bL}}\right)^{1/2} + \frac{\Delta p_{bG}}{\Delta p_{bL}}, \qquad [14]$$

which can be transformed (Chisholm 1973). to

$$\frac{\Delta p_b}{\Delta p_{bLO}} = 1 + (\Gamma^2 - 1)\{Bx^{2-n/2}(1-x)^{2-n/2} + x^{2-n}\}, \qquad [15]$$

where

$$\Gamma^2 = \frac{\Delta p_{bGO}}{\Delta p_{bLO}} = \frac{k_{GO}}{k_{LO}}\frac{\Delta \rho_L}{\rho_G} = \frac{\rho_L}{\rho_G}\left(\frac{\mu_G}{\mu_L}\right)^n, \qquad [16]$$

and approximately

$$C = B\Gamma. \qquad [17]$$

In Fitzsimmon's tests, the resistance coefficient k was independent of Reynolds number (Re ~ 10^6), whereas in Sekoda's tests k was slightly a function of Re($n = 0.08$). Where k is independent of Re, [15] reduces to [10].

For an air–water mixture at 1.5 bar with $n = 0.08$

$$\Gamma = (560)^{1/2}/(56)^{0.04} \doteqdot 20. \qquad [18]$$

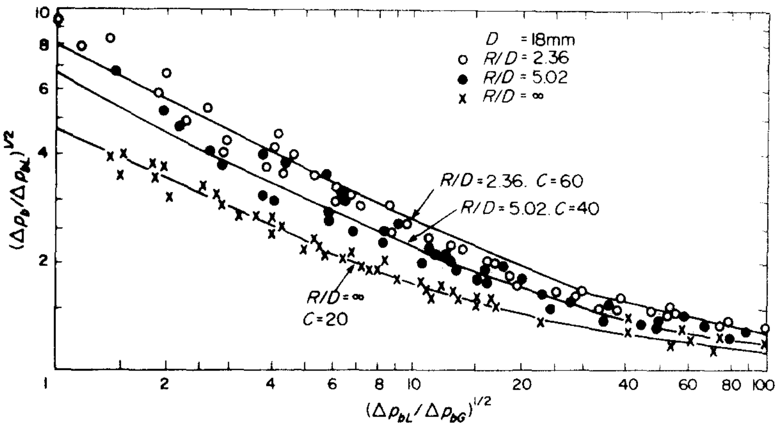

Figure 1. The root of the two-phase multiplier for 90° bends to a base of the Lockhart-Martinelli parameter.

Using [13] and [17] gives therefore for Sekoda's tests

$$R/D = 2.36 \quad C = 60$$
$$R/D = 5.02 \quad C = 40$$
$$R/D = \alpha \quad C = 20.$$

For simplicity the variation of n with R/D is ignored here. Figure 1 shows that [14] with these values of the coefficient C give good agreement with experiment.

This tends to confirm that [13] can be used at high density ratios, though consideration of the model does not necessarily suggest this. While Sekoda's analysis has necessitated consideration of the dependence of k on Re, this is a refinement not yet justified in practice; assume in practice $n = 0$ and $k_{GO} = k_{LO}$.

6. BENDS OTHER THAN 90° BENDS IN THE HORIZONTAL PLANE

Little evidence is available of the effect of the plane of the bend. Data of Peshkin (1961) for flow in rectangular channels with a horizontal inlet and vertical outlet showed little difference between downward and upward flow at outlet. The present method is recommended meanwhile with all geometries.

For bends other than 90°, pending experimental confirmation, our recommendation is to use [13] to evaluate the coefficient B. With a 180° bend for example, as k will be larger for the same R/D thant the 90°bend, both B and the two-phase multiplier will be lower than for the 90° bend. This is consistent with trends observed with tests on 90 and 180° bends using gas–solid mixtures (Uematsu 1964).

7. CONCLUSIONS

It has been demonstrated that, at least for the available data, the two-phase multiplier for a 90° bend can be evaluated using [10] with the coefficient B evaluated from

$$B = 1 + \frac{2.2}{k_{LO}\left(2 + \frac{R}{D}\right)}. \qquad [13]$$

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